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Case study: Risk analysis by overtopping of diversion works during dam construction: The *La Yesca* hydroelectric project, Mexico

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ABSTRACT

A risk analysis-based methodology for the determination of the most economical layout dam–tunnel diversion works is introduced. The aim of the proposed procedure is to identify the least cost layout in terms of the diversion works overtopping risk. The methodology has been built upon the reliability theory advanced first-order second moment approach, and accounts for the probability of the maximum height reached by the upstream water elevation, associated with a design flood (as characterized by its return period), as well as for excavation and lining costs. The proposed procedure has been applied to the *La Yesca* hydroelectric project in Mexico, currently under operation. It is demonstrated that the use of composite roughness, which consists of lining the floor of the diversion tunnels with hydraulic concrete, while the walls and vault of the tunnels are lined with shotcrete, results in an increase in the discharge capacity of the tunnels, thus leading to a reduction of the overall risk of the project. The importance of economic risk assessments is emphasized and the flexibility of the proposed methodology to account for a suite of risk–cost combinations is shown.

1. Introduction

Dam risk analysis has been a topic of much interest, particularly in Australia, Canada, The Netherlands, South Africa, USA and Mexico [1,7,8,17,20,16,3,2,10,15,21].

Statistical data from historic events are of limited utility in risk analysis. The shortcomings of such data have prompted the establishment of new databases, such as the one contained in the US National Performance of Dams Program Report [15], which provides much more reliable estimates of risk.

Marengo and Morales [14], analyzed the risk of failure of the *El Cajón* dam, Mexico, diversion works, and identified significant advantages of employing an economic risk analysis approach.

This paper is organized as follows. First, safety considerations for temporary works during dam construction are presented. Next, a brief description of risk assessment considerations on dam safety are commented and is made the description of La Yesca Hydroelectric Project including the description of risk conditions analysis for the diversion works, like the construction program, the potential damages in case of overtopping, and expected costs due to overtopping failure, are presented. Subsequently, a performance function for the analysis of risk is derived. Thereon, a sensitivity analysis is proposed. The methodology is applied to the case study of the *La Yesca* project, currently under operation in Mexico, with the objective of determining the optimal coffer dam height and the diversion tunnels sizing. Conclusions and recommendations are finally offered.

2. Safety considerations for temporary works during construction

Of 107 catastrophic dam failures worldwide, 61 occurred due to overtopping, and 13 of those cases occurred during construction [10]. On the basis of an analysis of these failures Marengo [12] concluded that "the design return periods required to ensure consistent safety levels in diversion works should have been higher by roughly a factor of ten".

Little attention has been devoted to dam safety during construction [4]. This lack of attention can be predominantly attributed to the following factors:

(a) Safety has traditionally been analyzed by considering only the damage likely to occur downstream from the dam under construction, without any consideration of the damage caused to the structures themselves or of the loss of revenue due to the delay in commencing power generation. Safety has been treated as a contractor responsibility, regardless of the consequences.

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(b) It is generally believed that a large flood is not likely to occur within the (usually short) construction period. However, available hydrologic evidence demonstrates that many destructive floods have occurred during large dam construction (e.g., at *Kariba*, *Oros*, *Aldedavilla*, *Akosombo*, *Cahora Bassa*, *Tarbela*, and *Aguamilpa*). The main lessons of *Aguamilpa's* overtopping were: risk assessment must take into account the specific features of each dam in more detail, and since the causes and consequences of failure are not easily predicted, there is a high degree of uncertainty that ought to be properly handled. Economic risk analysis provides an appropriate framework for analyzing safety during dam construction. Such analysis requires a sequential and conceptually concise approach, with accounts for each phase of the project.

In the *La Yesca* hydroelectric project, analyzed in this paper, the hydrologic risk of the dam construction (including diversion works), is taken by the owner, that is to say, the Federal Electricity Commission (CFE). Thus, the point of view of the owner of the dam is assumed in this paper. In other cases the construction contractor takes the hydrologic risk and some of the conclusions may differ.

3. Risk assessment of dam safety

Assessing the safety of dams requires an analysis of the effects of hypothetical failures and thus of the costs and benefits inherent to project safety.

A full dam failure analysis values the damage caused by lost services, estimates the construction costs of various design alternatives, determines the probability of failure for each alternative, and enables the selection of the design with the lowest risk–cost combination.

In particular, this approach depends on an accurate assessment of the potential risk presented by dam failure. Here, the term "risk" specifically refers to the total annual probability of failure multiplied by the cost of the consequences induced by this failure, including the partial or total loss of water storage at the time of the failure [11].

In addition, a complete risk assessment considers all possible events that could lead to dam failure. The partial risk of each individual event trajectory is equal to the product of the total annual probability of failure of the trajectory event or situation multiplied by the respective magnitude of the consequences of failure valued in monetary terms. By summing the partial risks, the total annual risk for the dam can be obtained as [9]:

$$R = \sum_i P_i C_i \tag{1}$$

where R is the total annual risk of failure of the dam; P_i, the total annual probability of failure for each situation or event i, and C_i is the cost of failure associated with situation or event i. In this study, R represents the actual risk is defined as the expected cost of the failure in the period of the analysis.

For the return period-based approach, the United States Army Corps of Engineers [19] states that the probability of a long-term failure can be estimated as follows:

$$P_f = 1 - \left(1 - \frac{1}{T_r}\right)^N \tag{2}$$

where P_f is the probability of failure, T_r is the return period of the design flood in years, and N is the period of analysis in years. For temporary works, the value of N is small (one or two years).

4. La Yesca hydroelectric project

The La Yesca hydroelectric project is under operation by the Federal Commission of Electricity (CFE), on the border of the Jalisco and Nayarit states in Mexico. The project, with a rockfill dam height of 208.50 m and a dam volume of 12 million m^3, is the second highest concrete face rockfill dam in the world, after *Shibuya* Dam in China.

The layout has an underground hydropower plant with two units, each of 375 MW capacity; such that the total installed power capacity is 750 MW. The annual mean power generation rate is 1228 GWh.

The diversion works were designed using a 50 year return period peak flow rate entrance of 6481 m^3/s, estimated on the basis of the 1953–2003 hydrological record for the dam site. These works comprise two tunnels with lengths of 703 m and 755 m, each with a portal cross-section of 14 $\times$ 14 m.

The coffer dam was built as a 48.5 m high earth and rock structure with an elevation of 435 masl (meters above sea level); Fig. 1 shows the dam, the coffer dam, and the cross section adopted for the tunnels).

Table 1
The maximum discharges associated with various return periods used in the design of the diversion works.

Return period (T_r)	Discharge (m^3/s)
10	3686
20	4958
50	6481
100	7578

4.1. Design floods for the diversion works

Given the hydrologic record for the period (1953–2003), the following probability distribution functions were fitted to the available data: normal, log-normal, exponential, Gamma, Gumbel, and Gumbel for two populations.

The probability distribution that provided the smallest square error was the Gumbel function for two populations and the values of various discharges and their corresponding return periods estimated with such function are shown in Table 1.

4.2. Risk conditions for a deterministic analysis of the diversion works

The risk conditions associated with the return period-based design of the diversion works were calculated for the first year of the project construction because it was assumed that after the second year the dam would achieve sufficient elevation that there would be no further significant risks of overtopping such as occurred in *Aguamilpa* in 1992 [12].

4.3. Diversion works (construction features).

With the goal of determining the optimal combination of the height of the coffer dam and the dimensions of the tunnel cross-section, a flood-routing analysis of the discharges associated with return periods of 20, 50, and 100 years was performed, and the total cost of the diversion works for each cross-section was found. The hydrograph form of the historical maximum flood of August 1973 (Fig. 2) was adopted and every flood was scaled with the discharges associated with the above mentioned return periods.

To determine the optimum height of the coffer dam, it was analyzed the discharge–elevation curves of the tunnel, and the following factors:

- The magnitudes of the expected floods associated with the calculated return periods.
- The construction program of the works. At the beginning of construction, a very important limiting factor is the fact that there are not enough roads, and neither is there sufficient machinery support. Because of this, an upper bound for the volume of earth materials to be placed at coffer dam was set at 700,000 m^3.

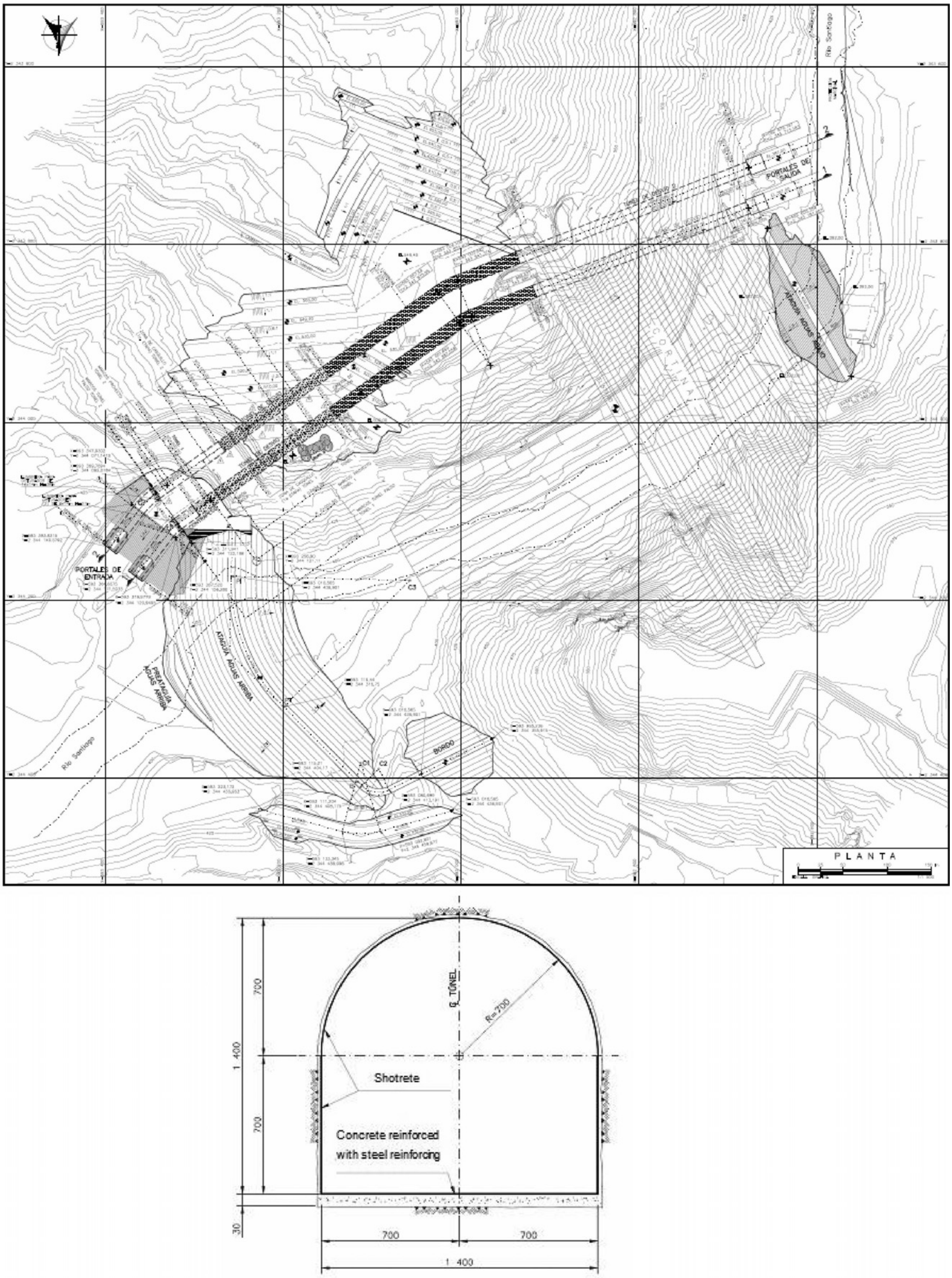

Fig. 1. Plan view and cross section of diversion tunnels of the La Yesca project.

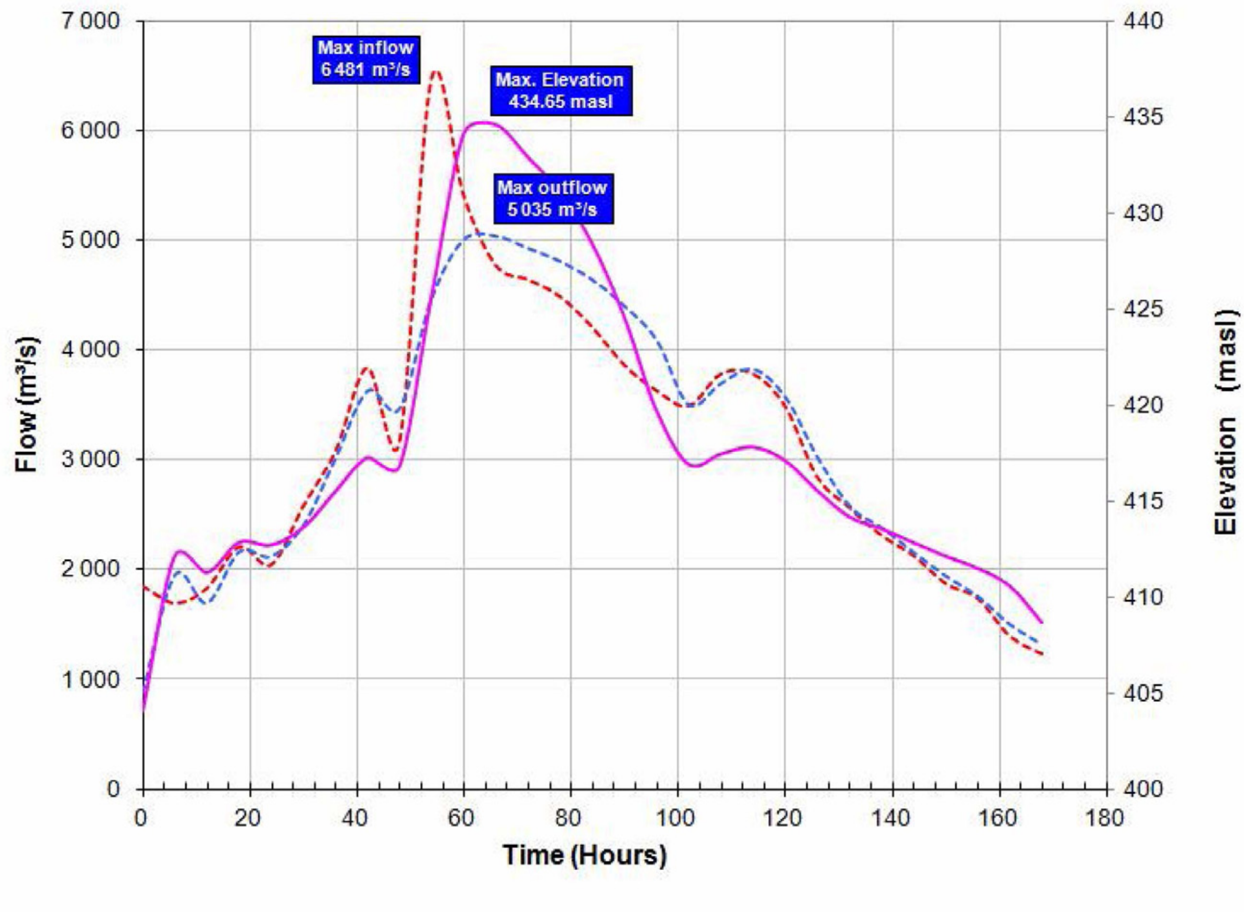

Fig. 2. Design flood with $T_r = 50$ years for La Yesca diversion works.

- The total cost of the structures involved (tunnels, coffer dams, and temporary and final closure structures of the diversion), given in average prices for 2005.
- The cost of late commencement of operation of the project plus the cost of the expected damage in the case of failures of the dam and coffer dams (i.e., the actual risk, CFE, 2005).
- The total project cost in the case of a failure due to coffer dam overtopping.

Analyses using tunnels with heights over 14.0 m were discarded because of the high cost of such tunnels.

The coffer dam cost did not include the acquisition of materials, as these were extracted from the excavation of the diversion tunnel. Table 2 shows the cost of different size of tunnels, the cost of the coffer dams (including the one located downstream), and the total cost of the diversion works for each alternative analyzed.

The height of the coffer dams were defined on the basis of geologic considerations, taking into account the location of a rock mass of acceptable quality on the left bank of the river [5]. The tunnels were built with a 14 $\times$ 14 m portal cross section and applying the concept of composite roughness [12], which consists of placing hydraulic concrete on the bottom and shotcrete on the walls and vault. The tunnel 1 has a length of 703 m and the tunnel 2, of 755 m respectively. The construction of the coffer dam involved using graded materials with an impervious core to achieve a volume of 600,000 m^3, and the height of the coffer dam and tunnels meant that the diversion works had a discharge capacity of 6481 m^3/s peak entrance flow.

There were lower-cost alternatives, but the risk of using, for instance, a 20-year design return period would have been unacceptably high, considering the size and importance of the works.

4.4. Construction program

The construction program for the analysis of La Yesca project, specified that the first tunnel would be completed on October 31, 2008, that work on the upstream coffer dam would commence on November 7 of that year, and that the second tunnel would be finished on January 15, 2009. The construction of the coffer dam was expected to be completed in late April of that year.

By considering various coffer dam heights, the volume to be placed, the construction time in months, and the completion date, it was projected that the construction of the diversion works as a whole, including the layers of soil injection at the bottom of both coffer dams, should be completed by July 1, 2009.

In the analysis presented herein it was assumed that at the beginning of construction it was feasible to place 130,000 m^3/month of earth materials in the coffer dam body. This assumption yielded the height, volume, placement, construction time, and date of completion of the coffer dams (see Table 3).

4.5. Potential damage

The selection of the most appropriate size of the diversion works depends on the magnitude of the damage that could occur in the

Table 2
Total costs of the diversion works associated with various return periods calculated with the deterministic approach.

Return period (years)	Portal section (m × m)	Coffer dam height (m)	Tunnel cost (million USD)	Coffer dam cost (million USD)	Total cost (million USD)
20	12.5 × 12.5	47.50	14.68	2.60	17.28
	13.0 × 13.0	44.00	15.95	2.30	18.25
50	12.5 × 12.5	59.00	14.68	3.85	18.53
	13.0 × 13.0	55.00	15.95	3.42	19.37
	13.5 × 13.5	52.00	17.23	3.05	20.28
	14.0 × 14.0	48.50	18.49	2.69	21.18
100	13.5 × 13.5	60.00	17.2.3	3.96	21.19
	14.0 × 14.0	56.00	18.49	3.53	22.02

Table 3
Height, volume, construction time, and date of completion of the coffer dams.

Coffer dam height (m)	Volume (million m^3)	Construction time (months)	Cofferdams completion date (2007)
45.5	0.58	4.4	19 March
48.5	0.66	5.1	10 April
57.0	0.83	6.4	19 May
63.0	1.00	7.7	28 June

event of a failure by overtopping. Such damage would affect the commencement date for power generation and the construction of the coffer dam and/or dam, depending on the degree of completion, and would result in additional costs due to the suspension of the construction works.

In general, it is also necessary to estimate the damage caused downstream by the failure of the diversion works, which could affect populated areas, infrastructure, and even human lives. At *La Yesca*, these considerations were not significant because the large *El Cajón* and *Aguamilpa* reservoirs, which can be used to regulate flow, are located immediately downstream of the dam site.

The potential damage was calculated according to the following considerations:

(a) Damage due to a delay of one year. The energy generated by the *La Yesca* dam will be 1228 GWh per year, with an estimated value of 108.67 million USD; these values were obtained by considering the peak, intermediate, and base components of the energy system. However, Mexico has a very high level of electrical grid power, so this cost is not seen as relevant because, in the event of a failure, another source of power generation would be available.
(b) Damage due to suspension of the work. These costs were estimated by assuming that an overtopping would cause a one-year construction delay. Several costs were considered [1]: the additional construction equipment costs (2.814 million USD), the overhead cost of the contractor (16.31 million USD) and additional funding required for a further year of construction (24.42 million USD), which total is 43.546 million USD (value used in the analysis).
(c) Damage of the coffer dam. Depending on the height and volume of the coffer dam, the unit price of materials, and the placement of the coffer dam and dam, the cost was USD 13.71/m^3, as shown in Table 4, which shows the height of the coffer dam, the crown's elevation, the volume, and the cost of reconstruction (the cost includes the procurement and placement of materials).
(d) Dam damage. This damage was assumed to be important only if failure occurred during the first year of construction, as overtopping would destroy the coffer dam and the dam. Failure during this period was assumed to destroy all progress made on both structures. From the second year on, it was assumed that 7.20 million m^3 of earth materials had been placed at the dam, reaching an elevation of 467 masl (more than any coffer dam selected). Once this material was in place, it was assumed that even an extraordinary flood would have no significant effect on the project.

The elevation of the dam, the volume placed in the dam, and the associated costs for the years 2008, 2009, and 2010 are shown in Table 5.

4.6. Expected damage costs

The expected damage cost is obtained by multiplying the probability of failure by the individual costs associated with a given failure probability and summing up the results, (see Eq. (1)).

For the dam itself, it was assumed that if what had been constructed were lost after one year (when the dam would have reached 40.50 m in height, corresponding to 426.00 masl in elevation and a volume of 1.51 million m^3) cost of 20.7 million USD would be at risk during the first year. This cost was held constant for estimating the dam failure risk, since in the second and third year of construction the dam would be high enough so that no significant dam loss risk would be entailed. The expected damage costs associated with delay and the dam loss in the first year are shown in Table 6.

If the coffer dam were destroyed, it would have to be entirely rebuilt; Table 7 shows the return period, the probability of failure, the coffer dam height, the cost of reconstruction, and the coffer dam destruction risk for this situation.

If failure due to overtopping occurs, the diversion tunnels are not affected and there is not cost by this effect.

Table 8 shows the risk costs of each aspect of overtopping (suspension, destruction of coffer dams, and dam), and the total risk cost, associated with different combinations of return period, tunnel cross-section and original coffer dam height.

According to this analysis, the combination with the lowest total cost corresponds to a return period of 50 years for construction of a coffer dam height of 48.50 m and two tunnels each with a portal cross-section of 14 × 14 m (4.156 MUSD). The next lowest cost involves a return period of 100 years for tunnels with the same cross-section and a 56 m high coffer dam, and the third lowest cost is for a return period of 50 years, a 13.5 × 13.5 m tunnel cross-section and a coffer dam height of 52 m.

The optimal combination determined by the return period-based analysis is in agreement with the decision made on the basis of the risk analysis.

Table 4
The cost of rebuilding coffer dams.

Cofferdam height (m)	Coffer dam crowns elevation (masl)	Volume (million m^3)	Cost (million USD)
44.00	429.00	0.51	6.990
47.50	432.50	0.58	7.950
48.50	433.50	0.66	9.050
56.00	441.00	0.83	11.380
59.00	444.00	0.85	11.650
63.00	448.00	1.00	13.710

Table 5
The cost of damage to the dam and the volumes associated with various elevations according to the year of construction.

Construction year	Dam elevation (masl)	Volume (million m^3)	Cost (million USD)
2008	426	1.51	20.70
2009	467	7.20	98.71
2010	532	10.8	148.0

Table 6
Suspension risk associated with the first year of operation of the dam.

Return period (years)	Q_p (m^3/s)	Risk of delay		Failure risk of the dam
		P_f	C_i (million USD)	C_i (million USD)
20	4958	0.05	2.177	1.035
50	6481	0.02	0.871	0.414
100	7578	0.01	0.435	0.207

Table 7
Expected damage due to destruction of the coffer dam.

Return period (years)	P_f	Height (m)	Coffer dam reconstruction (million USD)	Coffer dam destruction risk (million USD)
20	0.05	47.50	7.95	0.398
	0.05	44.00	7.040	0.352
50	0.02	59.00	11.650	0.233
	0.02	55.00	11.300	0.226
	0.02	52.00	11.250	0.225
	0.02	48.50	9.050	0.181
100	0.01	60.00	11.800	0.118
	0.01	56.00	11.400	0.114

Table 8
Total costs of dam construction and of expected damage.

Return period (years)	Section (m x m)	Coffer dam height (m)	Coffer dam original cost (million USD)	Risk by suspension (million USD)	Coffer dams destruction risk (million USD)	Risk from dam failure (million USD)	Total risk (million USD)
20	12.5 × 12.5	47.50	2.60	2.177	0.398	1.035	6.21
	13.0 × 13.0	44.00	2.30	2.177	0.352	1.035	5.864
50	12.5 × 12.5	59.00	3.85	0.871	0.233	0.414	5.368
	13.0 × 13.0	55.00	3.42	0.871	0.226	0.414	4.931
	13.5 × 13.5	52.00	3.051	0.871	0.225	0.414	4.561
	14.0 × 14.0	48.50	2.69	0.871	0.181	0.414	4.156
100	13.5 × 13.5	60.00	3.96	0.435	0.118	0.207	4.720
	14.0 × 14.0	56.00	3.53	0.435	0.114	0.207	4.286

5. Analysis of risk due to overtopping failure

5.1. Reliability

The purpose of a civil design [18] is to provide a reasonable margin of safety to structures that have a resistance X, and to provide a structure that can withstand a load Y. In the case of hydraulic structures, such as diversion tunnels or spillways, the “resistance variable” is considered to be the hydraulic conveyance capacity, and the “load variable” the peak flow rate of volume of floods. Traditionally, safety is described in terms of the safety factor, expressed as $SF = X/Y$, or by the safety margin, $SM = X - Y$. As will be shown later on, in the case of hydraulic structures it is more natural to work with the safety margin.

The variables SF, SM, X, and Y are usually treated as simple deterministic variables. If the resistance or load variables are random, SF and SM also become random variables.

When the analysis is performed using stochastic variables, the results are expressed in terms of the reliability index β, which may be interpreted as a measure of how far away is the state of a structure from failure.

In the application of this approach to hydraulic works (diversion works and spillways), the reliability index β can be related to the probability of the safety margin (probability that $P(SM > 0)$. Specifically, the probability of safety can be written as $P_s = \Phi(\beta)$ where and $\Phi(\cdot)$ is the normal probability distribution.

This approach assumes that floods and the water discharge capacity of the tunnels can be treated as stochastic variables.

The main parameters that influence risk analysis in this case are the design peak discharge, Q_p, the composite Manning roughness coefficient of the lining, n_e, and the size of the actual tunnel excavation, b.

The reliability index β can be calculated by successive approximations by employing the First-Order Reliability Moment (FORM) method until a tolerance criterion is met. The partial derivatives of the performance function are recalculated in each iteration, as are as the mean and standard deviation of the design peak discharge Q_p, μ_Q^N and σ_Q^N, with an equivalent normal distribution [18]. In addition, the determination of β also allows identification of the most probable dimensional surface that can cause system.

The equivalent normal distribution for a non-normal variable may be obtained such that the cumulative probability as well as the probability density ordinate of the equivalent normal distribution are equal to those of the corresponding non-normal distribution at the appropriate point, x_i^*, on the failure surface.

Thus, equating the cumulative probabilities at the failure point x_i^*, yields:

$$\Phi\left(\frac{x_i^* - \mu_{x_i}^{\mathrm{No}}}{\sigma_{x_i}^{\mathrm{No}}}\right) = F_{x_i}\left(x_i^*\right) \tag{3}$$

where $\mu_{x_i}^{\mathrm{No}}$ and $\sigma_{x_i}^{\mathrm{No}}$ respectively are the mean and standard deviation of the equivalent normal distribution of x_i, $F_{x_i}(x_i^*)$ is the original cumulative distribution of x_i, and, as before, $\Phi(\cdot)$ is the standard cumulative normal distribution.

From Eq. (3):

$$\mu_{x_i}^{\mathrm{No}} = x_i^* - \sigma_{x_i}^{\mathrm{No}}\Phi^{-1}\left[F_{x_i}\left(x_i^*\right)\right] \tag{4}$$

Equating the corresponding probability density ordinates at x_i^* means that:

$$\frac{1}{\sigma_{x_i}^{\mathrm{No}}}\phi\left(\frac{x_i^* - \mu_{x_i}^{\mathrm{No}}}{\sigma_{x_i}^{\mathrm{No}}}\right) = f_{x_i}\left(x_i^*\right) \equiv F'_{x_i}\left(x_i^*\right) \tag{5}$$

where $\phi(\cdot) \equiv \Phi'(\cdot)$ is the probability density function of the standard normal distribution. From Eqs. (4) and (5) it is possible to obtain:

$$\sigma_{x_i}^{\mathrm{No}} = \frac{\phi\left\{\Phi^{-1}\left[F_{x_i}\left(x_i^*\right)\right]\right\}}{f_{x_i}\left(x_i^*\right)} \tag{6}$$

5.2. Performance function

For the case of a dam or a coffer dam [9], the performance function $G(x)$ can be expressed in terms of the safety margin as

$$G(x) = H_p - H_g \tag{7}$$

where H_p is the elevation of the coffer dam or the dam (a fixed value) and H_g is the maximum water elevation reached in the reservoir when the design flood is routed through it (Fig. 3). The original hydraulic design of the tunnels in the diversion works was obtained by using the concept of composite roughness [13]. The hydraulic concrete lining Manning's roughness coefficient of the bottom of the tunnels was estimated to be $n_{cl} = 0.012$, where as the one corresponding to the shotcrete on the walls and vault, $n_{sc} = 0.025$. The equivalent roughness was estimated, based on a weighted average over the wetted perimeter, to have an equivalent value of $n_e = 0.02149$.

According to the construction site office of the CFE [6], the average over-excavation of the diversion tunnels along the perimeter of the cross section was 43 cm. Hydraulic concrete lining of 15 cm thickness was placed on the floor, whereas shotcrete lining of 15 cm thickness was placed in the walls and vault, leaving an actual cross section with average width and height of 14.56 m. Accordingly, the "as built" conditions of the tunnels (Fig. 4) show that the nominal section has an actual mean width and height of $b = 7.28$ m (total size of 14.56 m) with a standard deviation of 0.40 m, and an actual mean value for the roughness of $n_e = 0.0216$ and a standard deviation of 0.00015 (estimated from measurements made for Aguamilpa Case Study, [12]).

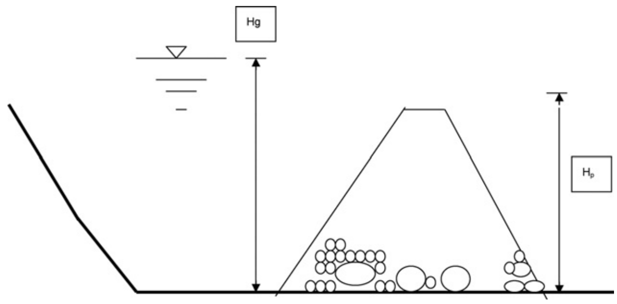

Fig. 3. Performance function for overtopping of a dam or cofferdam: $G(x) = H_p - H_g$, where H_p = height of upstream cofferdam (a fixed value) and H_g = final water elevation (data adapted from [9]).

Fig. 4. Diversion tunnel of the La Yesca project before the operation stage.

5.3. Hydraulic behavior of the diversion works

The hydraulic tunnel behavior can be described by assuming that the diversion works are temporary structures. A hydraulic design implies the development of an energy-based equation between the entrance and the exit of the diversion tunnels that can be expressed as [12]:

$$H_g = 1.015D_i - \frac{0.20}{A_i}\sqrt{\frac{D_i}{g}}Q_i + \frac{Q_i^2}{2gA_i^2}\left(1 + \frac{2gn_e^2}{R_h^{4/3}}L_i + K_{c1} + K_{c2} + +K_e + K_{con} + K_{ran}\right) + E_{p1} \tag{8}$$

where H_g is the elevation of water in the reservoir in meters above sea level (masl), D_i (m) is the equivalent diameter, A_i is the area (m^2) of each cross-section, Q_i is the analyzed flow (m^3/s) in each tunnel, L is the length of each stretch of tunnel (m), n_e is the Manning friction factor, R_h is the hydraulic radius (m), K_{c1}, K_{c2}, K_e, K_{con}, and K_{ran} are dimensionless coefficients of local head losses, and E_p denotes the elevation of the bottom of the tunnels outlet.

Then, by setting

$$\lambda_i = \frac{0.20}{A_i}\sqrt{\frac{D_i}{g}}Q_i \tag{9}$$

Table 9
FORM method results for various coffer dam heights.

Half section (m)	Coffer dam elevation (masl)	Coffer dam height (m)	P-FORM	T_r (years)	β	Q_p (m^3/s)	n_c	b (m)
7.0	420.00	33.50	0.0484	20.659	1.6605	5026	0.0215	7.3225
	422.73	36.23	0.03792	26.3713	1.7850	5478	0.02148	7.3250
	426.87	40.37	0.0251987	39.68	1.9567	6113	0.02151	7.3280
	435.00	48.50	0.0124894	80.076	2.24206	7223	0.0215	7.3334

Table 10
The results of the FORM method for non-failure probabilities of 0.95, 0.98 and 0.99.

Return period (years)	Portal section (m x m)	Coffer dam height (m)	Pure deterministic analysis	P_f FORM analysis	T_r FORM analysis (years)	Q (m^3/s)	n_c	b (m)
20	12.5 × 12.5	47.50	0.05	0.0287	34.82	5911	0.02146	6.524
	13.0 × 13.0	44.00	0.05	0.02872	34.814	5907	0.02142	6.797
50	12.5 × 12.5	59.00	0.02	0.0144	69.440	7010	0.0215	6.538
	13.0 × 13.0	55.00	0.02	0.01359	73.560	7090	0.02150	6.801
	13.5 × 13.5	52.00	0.02	0.0125	79.965	7225	0.0215	7.067
	14.0 × 14.0	48.50	0.02	0.012489	80.0676	7223	0.0215	7.3334
100	13.5 × 13.5	60.00	0.01	0.0070	109.71	7559	0.021	7.057
	14.0 × 14.0	56.00	0.01	0.006950	143.87	8132	0.0215	7.323

Table 11
Construction costs for failure probabilities of 0.05, 0.02 and 0.01 obtained with the FORM method.

Return period (years)	Portal section (m x m)	P_{FORM}	Coffer dam original cost (million USD)	Risk from suspension (million USD)	Coffer dam destruction risk (million USD)	Risk from dam failure (million USD)	Total risk (million USD)
20	12.5 × 12.5	0.0287	2.600	1.2498	0.2282	0.5941	4.6721
	13.0 × 13.0	0.02872	2.300	1.2506	0.2008	0.5945	4.3459
50	12.5 × 12.5	0.0144	3.850	0.6271	0.1687	0.2981	4.9439
	13.0 × 13.0	0.01359	3.420	0.5918	0.1547	0.2813	4.4478
	13.5 × 13.5	0.0125	3.050	0.5443	0.1277	0.2588	3.9808
	14.0 × 14.0	0.012489	2.690	0.5438	0.1130	0.2585	3.6053
100	13.5 × 13.5	0.0070	3.960	0.3048	0.0816	0.1449	4.4913
	14.0 × 14.0	0.006950	3.530	0.3026	0.0791	0.1439	4.0556

and

$$\psi_i = \frac{1}{A_i^2 2g}\left(1 + \frac{2gn^2 L_i}{R_h^{4/3}} + K_{c1} + K_{c2} + K_e + K_{con} + K_{ran}\right) \tag{10}$$

The following expression may be obtained:

$$H_g = 1.015D_i - \lambda_i Q_i + \psi_i Q_i^2 + E_{p1} \tag{11}$$

Eq. (8) should be applied to each of the tunnels under analysis, considering that $t + 1$ equations are available (where t is the number of tunnels).

For the 14 × 14 m diversion tunnels of *La Yesca* dam, the following equation was obtained by nested regression, employing Eq. (11), as well as the results of flood routing:

$$\begin{aligned} H_g = &\left(0.0000532b^{-2.73535} + 1.7073n^2b^{-4.06875}\right)Q_p^2 \\ &+\left(-0.002667b^{-3.562} + 0.01986b^{-1.780151}\right. \\ &\left.+605n^2b^{-3.113484}\right)Q_p \\ &+2.16398b - 0.4731b^{0.08151516} + 1.7613873b^{-0.82497} \\ &+52722.67338n^2b^{-2.158303} + E_{pi} \end{aligned} \tag{12}$$

where b is the half width of the tunnel (m) and E_{pi} denotes the elevation of the floor slab at the exit of each tunnel.

By considering coffer dams with various heights and an average width of 7.28 m, the parameters Q_p, ne, and b were obtained and are shown in Table 9.

The failure point result obtained for a coffer dam height of 48.50 m is: $Q_p = 7223$ m^3/s, $n_e = 0.0215$, $b = 7.3334$ m, $\beta = 2.24206$. The probability of satisfactory performance is: $P = \Phi(2.24206) = 0.9875106$. The probability of failure over a year of operation is: $P_f = 1 - 0.9875106 = 0.0124894$. The return period associated with this probability of failure is: $T_r = 80.0676$ years.

This analysis indicates a level of safety 60 percent that suggested by the deterministic analysis: the latter indicated a failure probability of 0.02, i.e. a return period $T_r = 50$ years compared to the return period of $T_r(0.012489) = 80.0676$ years obtained with the FORM method.

On the other hand, the discharge flow increases 11.45% with respect to the deterministic analysis and the width increase 4.78%. The roughness practically does not change.

5.4. Sensitivity analysis

In order to show the practical applications of the FORM method, it is presented a sensitivity analysis that considers tunnel cross-sections of 12.5 m, 13.0 m, 13.50 m, and 14.0 m and return periods of 20, 50, and 100 years.

Table 10 shows the original return period, the portal section, the coffer dam height, and return period-based failure probabilities. Also are shown the values corresponding to failure probabilities of the FORM method, his return periods and the values corresponding to the failure surface (discharge, roughness coefficient and the width).

The values obtained from the return period-based and FORM analyses show that, in all cases, the FORM method analysis is of the order of 43% higher than the obtained with the deterministic analysis, and in some cases is 60% higher.

This outcome is explained by the fact that the FORM analysis took into account the effects of the actual roughness, and width of the excavation, like the effects of reservoir regulation.

Return period-based designs are not conservative, so this suggests that when retrospective analyses are carried out it is possible to underestimate the discharges that cause failure in this type of construction.

By considering all the variables involved in the analysis, it is possible to obtain a better understanding of their combined behavior.

The original cost of construction of the coffer dam for various combinations of variables, the costs of risk due to suspension and destruction of the coffer dam and dam, and the total risk in millions of USD are shown in Table 11.

The minimum risk option for the diversion works involves a coffer dam height of 48.50 m and a tunnel cross-section of 14 × 14m (3.6053 million USD). However, as noted above, this option has a return period exceeding of 80.0676 years, and its level of safety is 60 percent higher than that obtained using the return period-based analysis.

Coincidently, like it was mentioned before, the risk analysis is in agreement with the based return period analysis, and was the decision adopted by the owner.

6. Conclusions

The following conclusions were obtained:

(1) The use of the FORM method to calculate the real failure probability of the system produces values of failure Q_p, n_e and b ($T_r =$ 80.0676 years) that are 60 percent higher than those obtained using the deterministic approach ($T_r = 50$ years).
(2) The FORM analysis provides a more realistic understanding of the conditions of the works by taking into account the actual width of the excavation, the final roughness obtained, and the regulation of the reservoir.
(3) Savings can be achieved in diversion works when tunnels are built with composite roughness because it is possible to decrease the number of tunnels (is improved the roughness) and increases the discharge capacity of the tunnels.
(4) Placing hydraulic concrete on the bottom and shotcrete on the vault and walls does not modify the original layout of the diversion works.
(5) In the Grijalva diversion tunnels (in Mexico), the concept of composite roughness is being used now.

Notation

The following symbols are used in this article:

b	half width of the of the tunnel
C_i	expected cost of failure for item i
$F(x)$	probability distribution function of a variable x
$G(x)$	reliability function of the variable (x)
H_g	water elevation at the end of the flood
H_p	elevation of the upstream coffer dam or dam.
N	evaluation period of the failure
$\mathrm{N_o}$	normal distribution notation
n_e	equivalent Manning roughness coefficient
P	probability
P_f	probability of failure
Q_p	peak discharge
SF	safety factor
SM	safety margin
T_r	return period
X	resistance variable
Y	load variable
β	reliability index
μ_Q^{No}	average value estimated with the equivalent normal distribution
σ_Q^{No}	standard deviation estimated cost with the equivalent normal distribution.

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